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D-003-001105

Seat No. ________

B. Sc. (Sem. I) (CBCS) Examination

March - 2022

Mathematics : Paper - M - 101(A)

(Geometry & Calculus) (Theory) (Old Course)

Faculty Code : 003

Subject Code : 001105

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instruction : All questions are compulsory.

1 Answer the following questions briefly : **20**

(1) Write Polar coordinates of (3, 4).

(2) Write relation between Cartesian and Polar coordinates.

(3) Write the polar form of equation $x^2 + y^2 = 6xy$.

(4) Define Sphere.

(5) Write Cauchy's mean value theorem.

(6) Define bounded set.

(7) Define least upper bound of a set.

(8) Define greatest lower bound of a set.

(9) Define differential equation.

(10) Solve : $ydx + xdy = 0$.

(11) Solve : $p^2 - 7p + 10 = 0$.

(12) Write Bernoulli's differential equation.

(13) Find the C.F. of $\left(D^2 + 2D + 1\right)y = 0$.

(14) Write expansion of $(1-D)^{-1}$.

(15) Write expansion of $(1+D)^{-1}$.

(16) Find $\frac{1}{D^2 - 9}e^x$.

(17) Evaluate : $\int_0^{\frac{\pi}{2}} \sin^4 x \, dx$.

(18) Evaluate : $\int_0^{\frac{\pi}{2}} \cos^3 x \, dx$.

(19) Write Reduction formula of $\int \sin^n x \, dx$.

(20) Evaluate : $\int_0^{\frac{\pi}{2}} \sin^5 x \, dx$.

2 (a) Attempt any **three** out of six : **6**

(1) Convert Cartesian equation $x^2 - y^2 = a^2$ in polar form.

(2) Find the equations of the sphere through the circle $x^2 + y^2 + z^2 = 9, 2x + 3y + 4z = 3$ and the point (1,1,1).

(3) If $y = (ax+b)^{-1}$, then prove that $y_n = \dfrac{(-1)^n n! a^n}{(ax+b)^{n+1}}$.

(4) Verify Rolle's theorem for $f(x) = x^2 - 3x + 2, x \in [1, 2]$.

(5) Evaluate : $\lim_{x \to 1} \dfrac{\log x}{x-1}$.

(6) Solve : $y = px + p - p^2$.

(b) Attempt any **three** out of six : **9**

(1) Find the distance between polar coordinates $\left(2, 10^0\right)$ and $\left(2, 20^0\right)$.

(2) Obtain radius and centre of the circle $r^2 - 8r\cos\left(\theta - \dfrac{\pi}{6}\right) + 12 = 0$.

(3) Evaluate : $\lim_{x \to 0} \dfrac{\log x^2}{\cot x^2}$.

(4) Evaluate : $\lim_{x \to a} \frac{\log(x-a)}{\log\left(e^x - e^a\right)}$.

(5) Find the sphere for which $A(2, -3, 4)$ and $(-2, 3, -4)$ are the extremities of a diameter.

(6) Obtain the equation of a circle whose centre is (ρ, α) and radius is a.

(c) Attempt any **two** out of five : **10**

(1) Find the equation of the sphere passing through the points $O(0,0,0), A(-a,b,c), B(a,-b,c)$ and $C(a,b,-c)$.

(2) State and prove Rolle's mean value theorem.

(3) State and prove Lagrange's mean value theorem.

(4) For $x > 0$ prove that $\frac{x}{1+x^2} < \tan^{-1} x < x$.

(5) Evaluate : $\lim_{x \to \frac{\pi}{2}} (\sin x)^{\tan x}$.

3 (a) Attempt any **three** out of six : **6**

(1) Solve : $y^2 + p^2 = 0$.

(2) Solve : $y + px = p^2 x^4$.

(3) Solve : $\left(D^2 - 3D - 4\right) y = 0$.

(4) Find $\frac{1}{D^3}\left(5x^2\right)$.

(5) Prove that $\frac{1}{D^2 + a^2} \cos ax = \frac{x}{2a} \sin ax$.

(6) Evaluate : $\int_0^{\frac{\pi}{4}} \sin^{16} 2x$.

(b) Attempt any **three** out of six : **9**

(1) Solve : $\left(D^3 - 7D - 6\right) y = 0$.

(2) Solve : $\left(D^2 - 7D + 12\right) y = e^{5x}$.

(3) Solve : $y = 2px + y^2 p^3$.

(4) Solve : $x^2 (y - px) = yp^2$.

(5) Evaluate : $\int_0^{\frac{\pi}{6}} \cos^6 3x$.

(6) Evaluate : $\int_0^{a} x^4 \left(a^2 - x^2\right)^{\frac{3}{2}} dx$.

(c) Attempt any **two** out of five : **10**

(1) Obtain the solution of linear differential equation $y' + Py = Q$.

(2) Obtain the solution of Bernoulli's differential equation $y' + Py = Qy^n$.

(3) Obtain Reduction formula for $\int \sin^n x \, dx, n \in N$.

(4) Evaluate : $\int_0^{\infty} \frac{dx}{\left(1 + x^2\right)^3}$.

(5) Solve : $\left(D^3 + D^2 - D - 1\right) y = \cos 2x$.
